WESTMINSTER SCHOOL

THE CHALLENGE 2014

MATHEMATICS III

Wednesday 30th April 2014

Time allowed: 1 hour 30 minutes

You may not use a calculator in this paper.

All your working should be clearly shown.

You should attempt all the questions.

1 Alfie, Ben and Charlie are brothers. They have a box which is $10\frac{3}{4}$ inches tall. When Alfie stands on the box, he is still $1\frac{1}{4}$ inches shorter than Charlie, but $4\frac{2}{3}$ inches taller than Ben. When Ben stands on the box, how much taller than Charlie will he be?

2 When I use fourteen of my gardeners to prune the 350 rose trees on my terrace it takes 15 days. How long would it take to prune the 480 rose trees in my walled garden if I used eighteen of my gardeners?

3 In a sequence of numbers, each number is followed by the *sum of the cubes of its digits*.
For example, in the sequence starting with 1124, the second number is

$$1^3+1^3+2^3+4^3=1+1+8+64=74,$$

the third number is

$$7^3+4^3=343+64=407$$

and the fourth number is

$$4^3+0^3+7^3=64+0+343=407$$

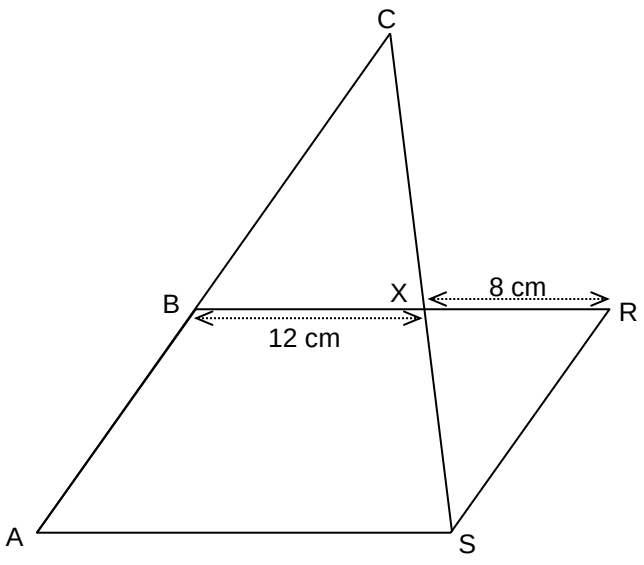
and so all the numbers in the sequence after this will also be 407.

Find the 100^{th} number in the sequence starting with each of the following numbers.

a 2521
b 217
c 793

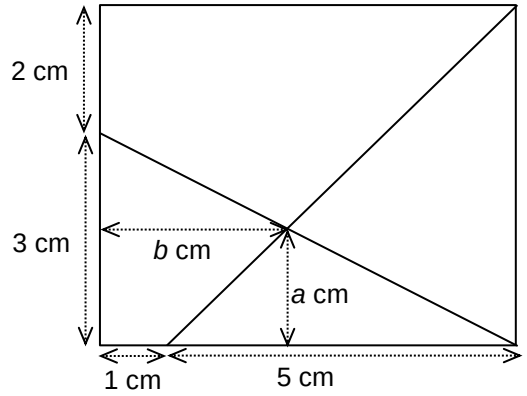
4 At a concert, where there are 60 seats in the audience, the organisers charge different amounts to adults, children and senior citizens.
If the tickets are sold to 24 adults, 20 children and 16 senior citizens, the organisers take £516.
If the tickets are sold to 15 adults, 29 children and 16 senior citizens, the organisers take £453.
If the tickets are sold to 30 adults, 20 children and 10 senior citizens, the organisers take £540.
What is the cost of an adult ticket?

5 In the diagram, ABRS is a parallelogram. The area of trapezium ABXS is 144 cm^2.



a Find the area of triangle XRS.
b Find the area of triangle BXC.

6 Use the information in the diagram below, which shows a rectangle and two straight lines, to find lengths *a* and *b*.



7 The diagram shows part of a very long strip of paper with numbers down the middle, and colours marked along each edge. The colours red (r), yellow (y) and green (g) are represented by letters.

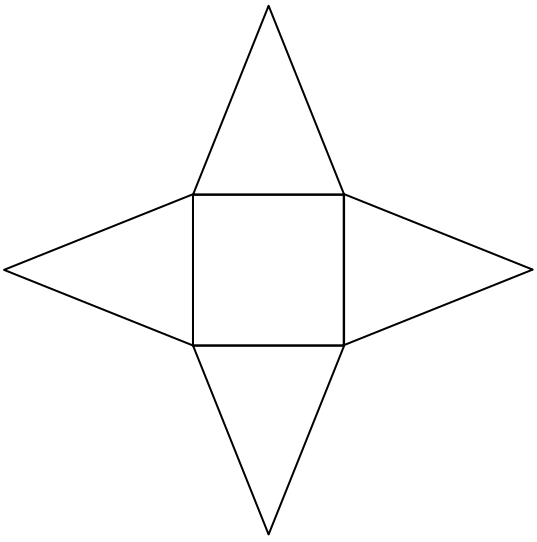
r	g	g	y	g	r	r	g	g	…
1	2	3	4	5	6	7	8	9	…
r	g	r	g	y	r	g	r	g	…

On the top edge there is a repeating pattern r g g y g r;
on the bottom edge there is a repeating pattern r g r g y.

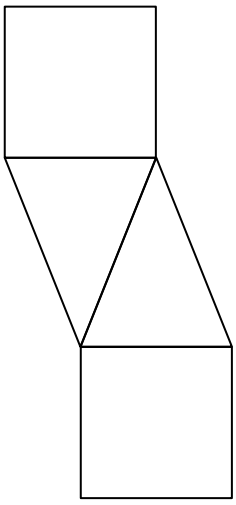
a The numbers 1 and 6 have red marked both above and below.
Which other numbers have red marked both above and below?

b If, instead, there was a repeating pattern, on the top edge, of y r g r g y g g
and, on the bottom edge, of r g y g r y,
which numbers would have red marked both above and below? Justify your answer.

8 The diagram shows two shapes, made up of a total of three identical squares and six identical isosceles triangles. The area of each shape is written underneath it.
Find the side-length of the square and the height of the triangle.



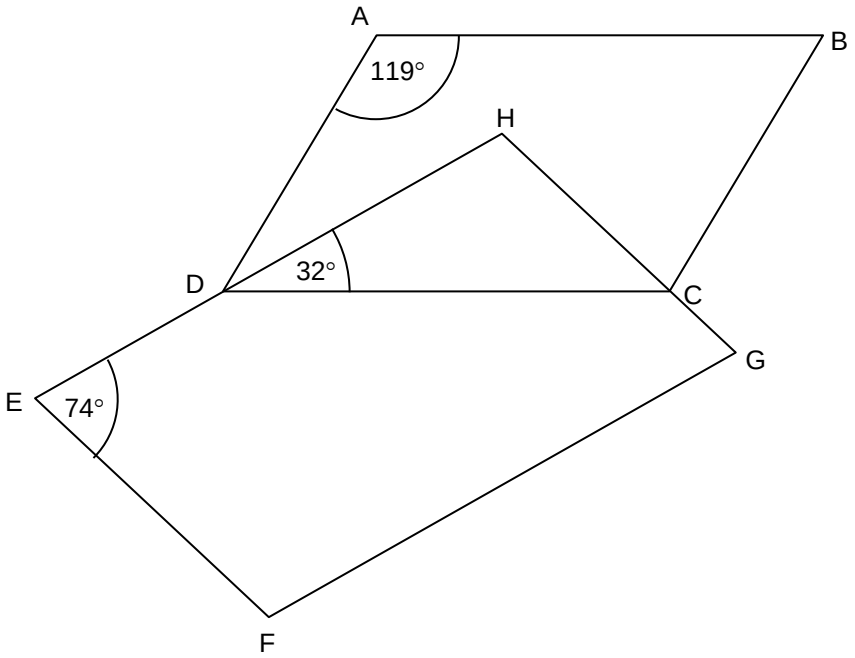
Area: 175 cm^2



Area: 161 cm^2

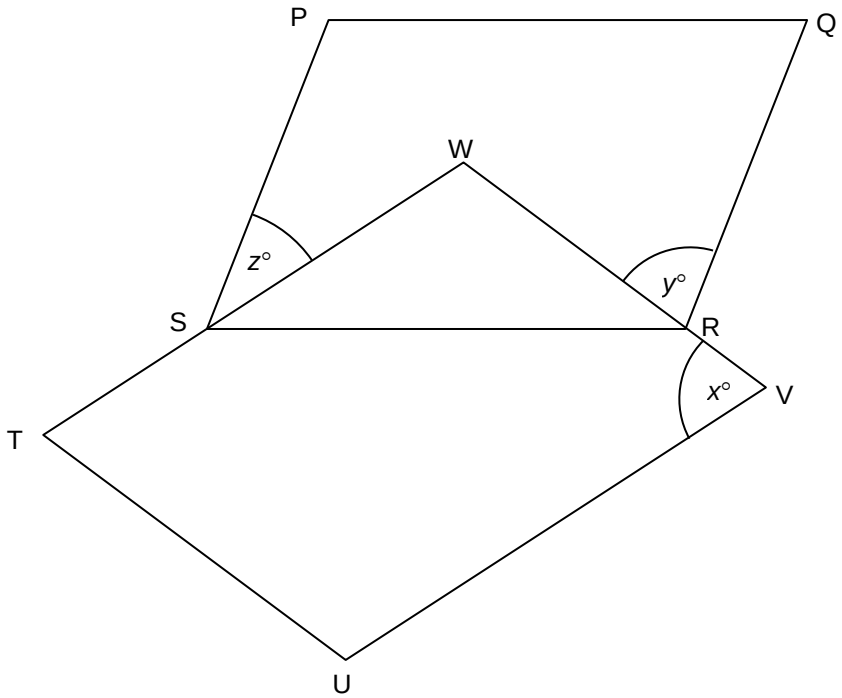
TURN OVER

9 **a** In this diagram, ABCD and EFGH are parallelograms.



Find angle BCH. Give a clear justification for your derivation.

b In this diagram, PQRS and TUVW are parallelograms.



Prove that $x + y + z = 180$.

10 Evan and Fred are running a race. Evan covers the distance at a constant speed of 6 kilometres per hour. Fred covers the first half of the distance at 7·5 kilometres per hour and the second half of the distance at a slower, but constant, speed. The race is a dead heat. How fast did Fred run over the second half of the distance?

11 A Tribonacci sequence is made up of positive whole numbers. In a Tribonacci sequence, the first three numbers are in increasing order, and each number in the sequence after the third is the sum of the three preceding numbers.
For example, a Tribonacci sequence could start with the numbers 1, 3, 11 (but not 3, 11, 1);
then the fourth number would be 1 + 3 + 11 = 15
and the fifth number would be 3 + 11 + 15 = 29.

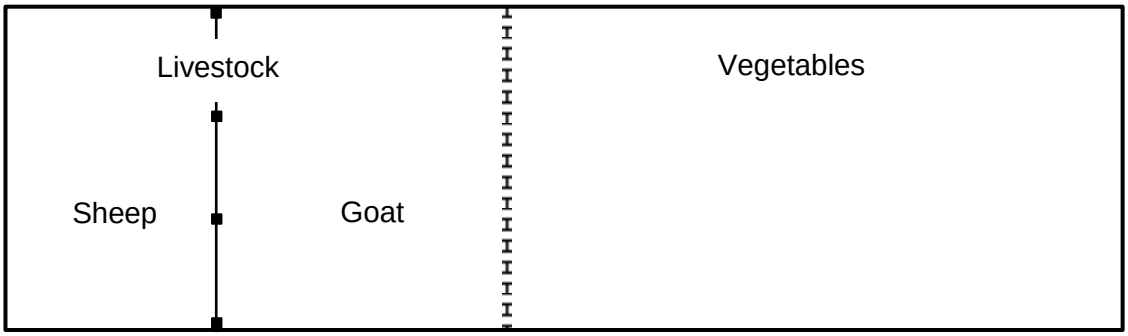
a If the first three numbers in a Tribonacci sequence are 1, 2, 3, find the 8^{th} number in the sequence.

b For the Tribonacci sequence whose first five terms are shown below, find x and y.
2 x 9 y 30 ….

c **i** If the first three numbers in a Tribonacci sequence are a, b, c, write the 6^{th} number in the sequence in terms of a, b and c.
ii Find the Tribonacci sequence whose 7^{th} term is 77.

12 The diagram below shows my rectangular garden. A wall divides it into a part for livestock and a part for growing vegetables. The part for growing livestock is divided by a fence into a part where I keep my goat, and a part where I keep my sheep.

a The part where I keep my sheep is 75% of the size of the part where I keep my goat. The part where I grow vegetables is 65% of the whole garden. What percentage of the whole garden is the part where I keep my goat?



b Next year, I will redesign my garden by moving the wall and the fence. The part devoted to vegetables will decrease in size by 20% and the part where I keep my sheep will increase in size by 40%. By what percentage does the part where I keep my goat increase in size?

13 **a** A square and a rectangle have the same area. The rectangle has a height which is 6 cm greater than the side of the square, and a base which is 4 cm less than the side of the square. What is the area of the square?

b Three rectangles have the same area.
The second rectangle has a height which is 4 cm greater than the height of the first, and a base which is 1 cm less than the base of the first.
The third rectangle has a height which is 5 cm less than the height of the first, and a base which is 2 cm greater than the base of the first.
What is the area of the rectangles?

TURN OVER

14 Four points, A, B, C and D, are at the vertices of a square. The point X is a distance of 5 cm from point A, 10 cm from point B and 14 cm from point C. How far is X from point D?
The dotted lines are drawn to help you think about the problem.