WESTMINSTER SCHOOL

THE CHALLENGE 2014

MATHEMATICS II

Tuesday 29th April 2014

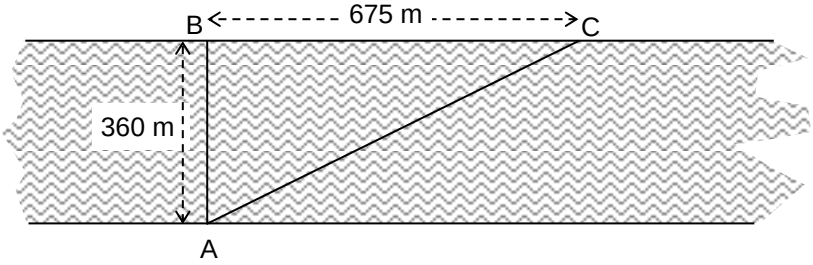
Time allowed: 1 hour 30 minutes

You will need a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

1 Alex is at point A on one side of a river, and wants to get to point C.



Alex can row at 4·5 metres per second.

a How long does he take to row in a straight line from A to C?

Alex finds that if, instead, he rows directly across the river to point B, and then runs along the bank from B to C, he takes the same total length of time to get from A to C.

b How fast does Alex run?

2 **a** What is the difference between $a+b+c$ and $a+b-c$?

b Divide x^2 by $\frac{x}{2}$, and simplify your answer.

c Make T the subject of the formula $\frac{a}{T+1}=b$.

d Solve the equation

$$\frac{x}{5}+1=\frac{x}{3}.$$

3 **a** The length of a rectangle is 40% larger than its width.
The area of the rectangle is 315 cm^2.
What is its perimeter?

b The length of a rectangle is 40% larger than its width.
The diagonals of the rectangle are 21·5 cm long.
What is its area?

4 **a** Solve the simultaneous equations

$$5x-3y=11$$

$$2x+\tfrac{1}{2}y=18$$

b In the simultaneous equations below, one of the numbers has been blanked out.

$$2x+3y=11$$

$$\square x-5y=5$$

Sam has solved the simultaneous equations correctly. One of the numbers in his solution has been blanked out

$x=\bigcirc$ and $y=2$

Work out the two numbers that have been blanked out.

5 In Ireland, VAT is charged at a rate of 23%. One Euro is worth 82 p.
A computer ordered from an Irish company costs €891, with Irish VAT included.
If you did not have to pay Irish VAT, what would this computer cost in pounds?

6 Arthur has forty cup-cakes. He puts 30 grams of icing on each of ***n*** of the cakes, but begins to run out of icing and can only put 20 grams of icing on half of the remaining cakes and 10 grams of icing on the other half of the remaining cakes.

a Write an expression for the amount of icing he used.

In fact, Arthur used 870 grams of icing.

b How many of the cakes had 30 grams of icing on?

7 When you order footballs from a website, you are charged £4·80 each for them, unless you order 20 or more footballs, in which case you are charged £4·20 each for them.

a If you actually need 19 footballs, what percentage saving do you make by ordering 20 footballs instead of 19?

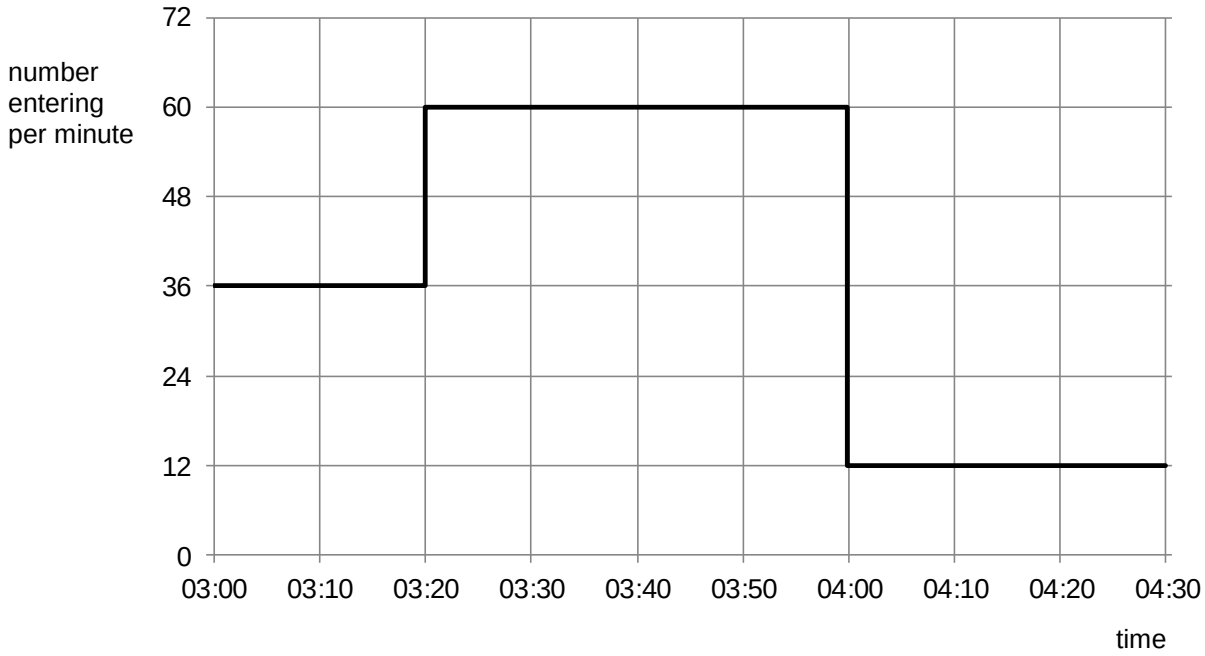
If you order 50 or more footballs from the website, you are only charged £3·60 each for them.

b For what numbers of footballs that you actually need would you save money by ordering 50 footballs?

If you order 100 or more footballs from the website, you are charged even less each for them. In fact, it is cheaper to order 100 footballs if you actually need 92 to 99 footballs, but not if you actually need 91 or fewer.

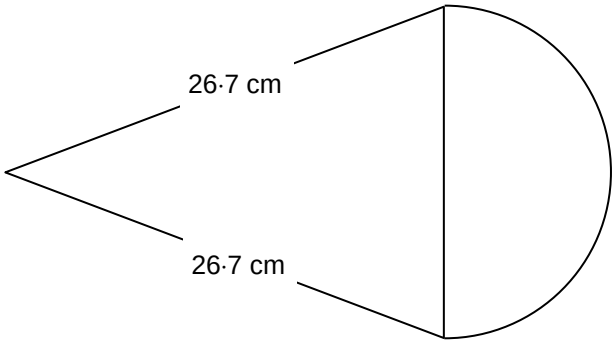
c What could be the price charged for each football if you order 100 or more?

8 The graph below shows the rate at which spectators are entering a stadium via one of the entrances, at different times after the gates are opened at 3:00.



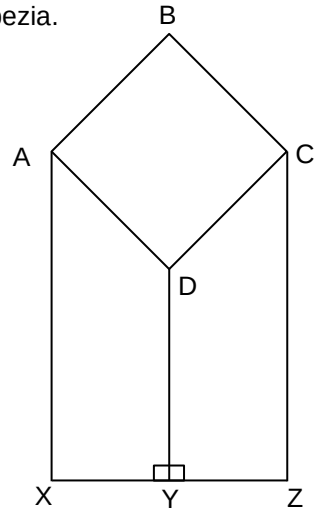
How many people enter the stadium by this entrance between 3:00 and 4:30?

9 The diagram shows a semicircle and an isosceles triangle. The area of the semicircle is 230 cm^2. What is the area of the triangle?



TURN OVER

10 The diagram shows a square ABCD and two identical trapezia ADYX and CDYZ.
The area of the square is 84·5 cm^2.
The length DY is 11·75 cm.
Find the area of one of the trapezia.



11 In 2011, and in 2013 (though not in 2012), Sam and Tom had the same amount of pocket money.
Tom's pocket money went up by 26% between 2011 and 2012. It also went up by 26% between 2012 and 2013. Sam's pocket money went up by 8% between 2011 and 2012.

a By what percentage did Sam's pocket money go up between 2012 and 2013?

In 2012, Tom's pocket money was £1·44 more than Sam's.

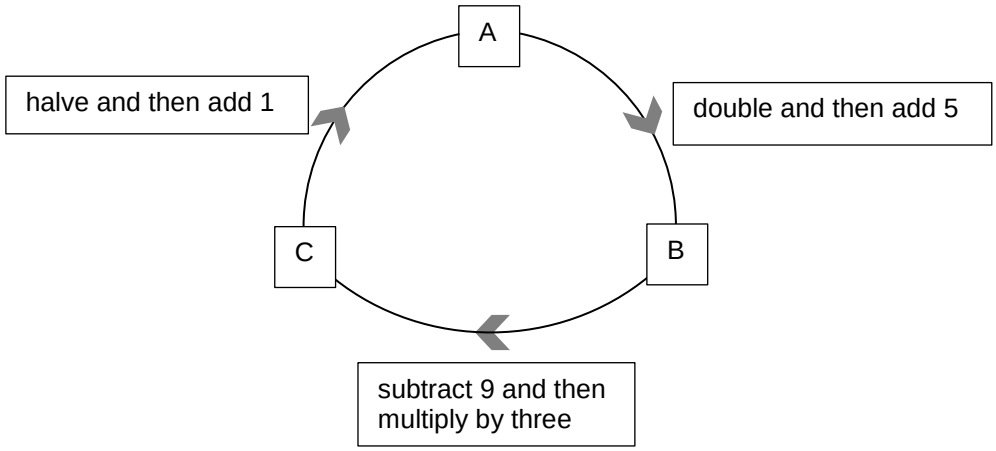
b How much pocket money did Sam and Tom get in 2011?

12 In a school, the classes are of different sizes, but they always consist of between 20 and 30 pupils.

a In one form, the mean average height of the ten boys is 156 cm and the mean average height of the fifteen girls is 161 cm. What is the mean average height of the whole class?

b In another form, the mean average height of the boys is 155 cm and the mean average height of the girls is 147 cm. The mean average height of the whole class is 152 cm.
How many boys and how many girls are there in the class?

13 The diagram below shows how three numbers are related. Find the three numbers.



14 To work out the area of a triangle with sides of length a, b and c:

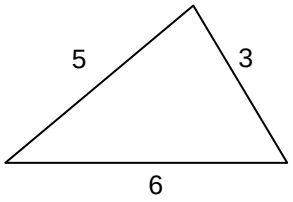
i calculate half the length of the perimeter of the triangle and call this s.

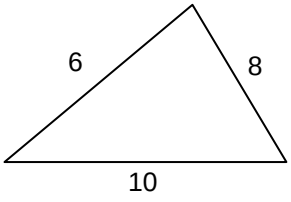
ii The area is given by Heron's formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

a Write a formula for s in terms of a, b and c.

b Use Heron's formula to work out the areas of the triangles shown below.





c There is an easier method for finding the area of the second triangle in **b**.
What is this method?

d Find the height, h, of the triangle below.