The King's School Canterbury

Sixth Form Entrance 2015

MATHEMATICS

1 hour

Attempt all questions if possible. Do not worry if there are topics you have never covered; do your best on whatever you can attempt.
Questions are not necessarily in order of difficulty.
Marks for parts of questions are given in brackets as a guide.
Show as much working as you can. Calculators are allowed and their use expected.
There is a list of formulae at the front, not all of which need necessarily be used in this paper.

The paper has twenty-four questions. Work quickly.

There are one hundred and fifteen marks in total.

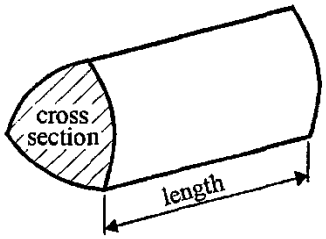
NAME: ..... AGE: .....

PRESENT SCHOOL: .....

Total:	**%**

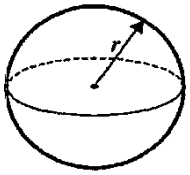
Formula Sheet

Volume of prism = area of cross-section × length



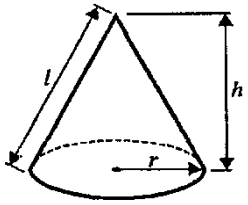
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

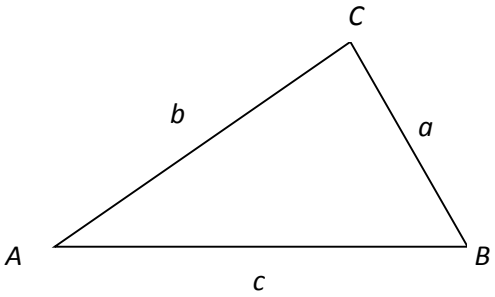


Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl



In any triangle *ABC*



Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of a triangle = $\frac{1}{2}ab\sin C$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Q1. Karen buys 19 identical calculators. The total cost is £143.64

Work out the total cost of 31 of these calculators.

£ .....

(Total 3 marks)

Q2. There are 40 litres of water in a barrel.

The water flows out of the barrel at a rate of 125 millilitres per second.

1 litre = 1000 millilitres.

Work out the time it takes for the barrel to empty completely.

..... seconds

(Total 3 marks)

Q3. In a sale all the normal prices are reduced by 18%.
In the sale Mandy pays £12.71 for a hat.

Calculate the normal price of the hat.

£.....

(Total 3 marks)

Q4. There are some sweets in a bag.

18 of the sweets are toffees.
12 of the sweets are mints.

(a) Write down the ratio of the number of toffees to the number of mints.
Give your ratio in its simplest form.

..... : .....

(2)

There are some oranges and apples in a box.

The total number of oranges and apples is 54
The ratio of the number of oranges to the number of apples is 1 : 5

(b) Work out the number of apples in the box.

.....

(2)

(Total 4 marks)

Q5. (a) Expand and simplify $(x - 17)(x - 4)$

…… **(2)**

(b) Solve $\dfrac{x - 17}{x - 4} = 5$

…… **(3)**

(Total 5 marks)

Q6. Solve the simultaneous equations

$$2x + 3y = 6$$
$$3x - 2y = 22$$

x = ……

y = ……

(Total 4 marks)

Q7. Simplify fully

(i) $m^4 \times m^5$

.....

(ii) $p^6 \div p^2$

.....

(iii) $5x^3y \times 2xy^8$

.....

(iv) $\dfrac{4(k+8)^2}{k+8}$

.....

(Total 6 marks)

Q8. A straight line has equation $y = 4x - 5$.

Write down the equation of the straight line that is parallel to $y = 4x - 5$ and passes through the point (0, 3).

.....

(Total 2 marks)

Q9. (a) Work out, giving your answers as **integers** or **fractions** as appropriate:

(i) 8^0

.....

(ii) 5^{-2}

.....

(iii) $25^{\frac{1}{2}}$

.....

(iv) $27^{-\frac{1}{3}}$

.....

(7)

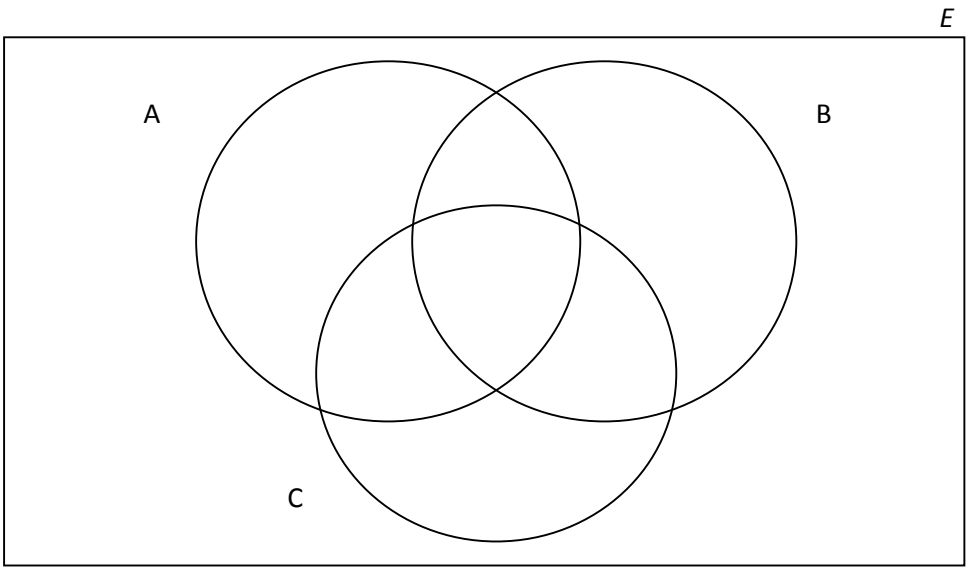
(b) Given that $x = 2^k$ and $\sqrt{\frac{4}{x}} = 2^c$, find c in terms of k.

c = ...

(3)

(Total 10 marks)

Q10. The diagram shows three intersecting sets A, B and C.



a) Lightly shade the region described by $(A \cap B) \cap C'$

(1)

In a school all of the 120 pupils must choose at least one of Archery, Badminton or Cycling.

63 pupils choose archery
62 pupils choose badminton
69 pupils choose cycling
28 pupils choose archery and badminton
27 pupils choose archery and cycling
32 pupils choose badminton and cycling.

b) Let the number of people who choose all three options be x

i) Write down in terms of x the number of people who choose archery and badminton but not cycling.

……**(1)**

ii) Show that the number of people who choose archery only is $8 + x$

……**(2)**

iii) Form and solve an equation to find x.

……**(3)**

(Total 7 marks)

Q11.

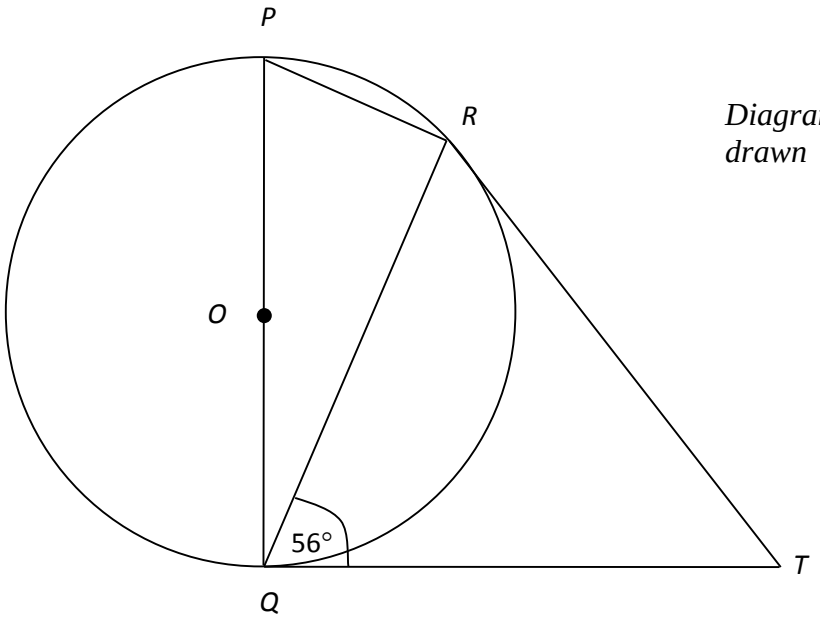


*Diagram **NOT** accurately drawn*

P, *Q* and *R* are points on a circle, centre *O*. *POQ* is a straight line.

TQ and *TR* are tangents to the circle.

Angle *TQR* = 56°.

(a) Explain why angle *PQR* = 34°.

……

……

(1)

(b) Calculate the size of angle *PRT*.
Give reasons for your answer.

…….°

(3)

(Total 4 marks)

Q12. y is directly proportional to the **square** of x.

When $x = 4$, $y = 25$.

(a) Find an expression for y in terms of x.

.....

(3)

(b) Calculate y when $x = 2$.

.....

(1)

(c) Calculate x when $y = 9$.

.....

(2)

(Total 6 marks)

Q13.

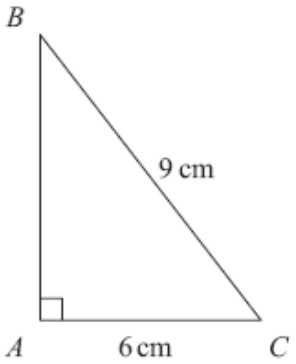


Diagram **NOT** accurately drawn

ABC is a right-angled triangle.
AC = 6 cm.
BC = 9 cm.

Work out the length of *AB*. Give your answer correct to 3 significant figures.

..... cm

(Total 3 marks)

Q14. Sethina recorded the times, in minutes, taken to repair 80 car tyres.
Information about these times is shown in the table.

Time(*t*minutes)	**Frequency**		
$0 < t \leq 6$	15		
$6 < t \leq 12$	25		
$12 < t \leq 18$	20		
$18 < t \leq 24$	12		
$24 < t \leq 30$	8		

Calculate an estimate for the mean time taken to repair each car tyre.

..... minutes

(Total 4 marks)

Q15. Consider the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

Given $u = 2\frac{1}{2}$, $v = 3\frac{1}{3}$

(a) Find the value of f **without a calculator and showing working**

.....

(3)

(b) Rearrange $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

to make u the subject of the formula.

Give your answer in its simplest form.

.....

(3)

(Total 6 marks)

Q16. Here is a right-angled triangle.

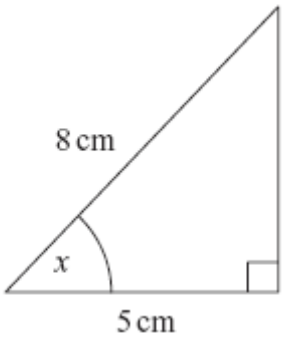


Diagram **NOT** accurately drawn

(a) Calculate the size of the angle marked *x*.
Give your answer correct to 1 decimal place.

x = ...

(3)

Here is another right-angled triangle.

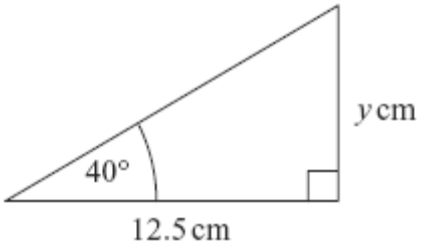


Diagram **NOT** accurately drawn

(b) Calculate the value of *y*.
Give your answer correct to 1 decimal place.

y = ...

(3)

(Total 6 marks)

Q17. Peter cuts a square out of a rectangular piece of metal.

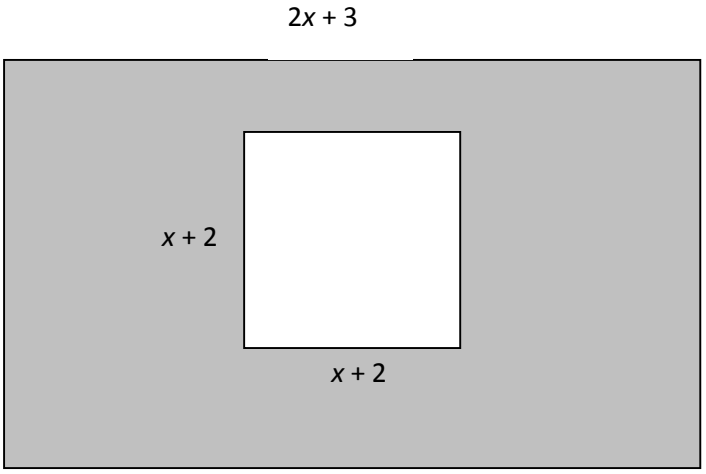


Diagram **NOT** accurately drawn

The length of the rectangle is $2x + 3$.

The width of the rectangle is $x + 4$.

The length of the side of the square is $x + 2$.
All measurements are in centimetres.

The shaded shape in the diagram shows the metal remaining.

The area of the shaded shape is 20 cm^2.

(a) Show that $x^2 + 7x - 12 = 0$

…… **(4)**

(b) (i) Solve the equation $x^2 + 7x - 12 = 0$

Give your answers correct to 4 significant figures.

.....

(3)

(ii) Hence find the perimeter of the **square**.

Give your answer correct to 3 significant figures.

..... cm

(1)

(Total 8 marks)

Q18. Phil has 20 sweets in a bag.

5 of the sweets are orange.
7 of the sweets are red.
8 of the sweets are yellow.

Phil takes at random **two** sweets from the bag.

Work out the probability that the sweets will **not** be the same colour.

.....

(Total 4 marks)

Q19. Simplify fully $\frac{x^2 - 8x + 15}{2x^2 - 7x - 15}$

.....

(Total 4 marks)

Q20. In the circle shown below, O is the centre and RV is a tangent touching the circle at point T. Angle ATR = 62° and angle BTV = 79°. Find the value of the angle *y*, giving a **reason** for each step of your working.

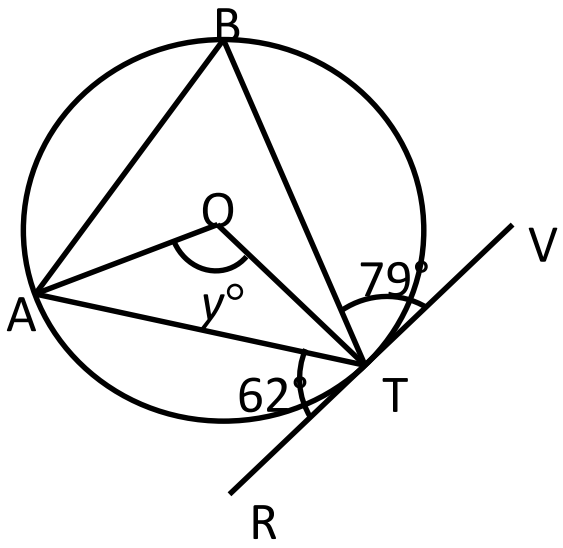


Diagram NOT accurately drawn

y = ... °

(Total 4 marks)

Q21. Solve simultaneously the equations

$y = x^2 - 3x + 7$ and

$y = 2x + 1$

……

(Total 5 marks)

Q22.

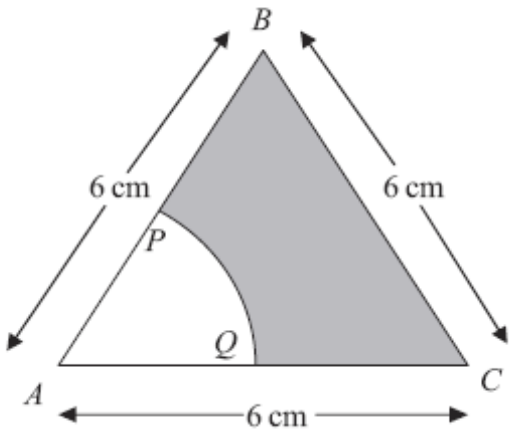


Diagram **NOT** accurately drawn

The diagram shows an equilateral triangle *ABC* with sides of length 6 cm.

P is the midpoint of *AB*.
Q is the midpoint of *AC*.
APQ is a sector of a circle, centre *A*.

Calculate the area of the shaded region.
Give your answer correct to 3 significant figures.

..... cm^2

(Total 4 marks)

Q23. If PQ = 7 km, PR = 11 km and angle QPR = 83°, work out length QR correct to 3 significant figures.

……

(Total 5 marks)

Q24. Prove that $(3n + 1)^2 – (3n –1)^2$ is a multiple of 4, for all positive integer values of n.

(Total 5 marks)

END OF EXAM

THE KING'S SCHOOL, CANTERBURY

SIXTH FORM ENTRANCE EXAMINATION

2014-2015

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Total:	**%**

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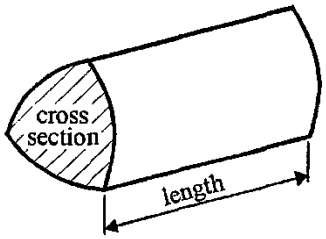

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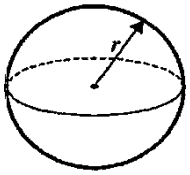

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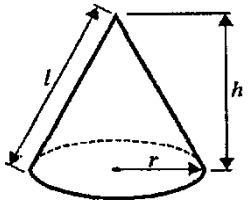
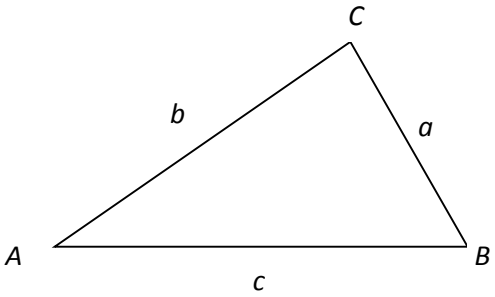


In any triangle *ABC*



Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of a triangle = $\frac{1}{2}ab\sin C$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Q1.

(a) (i) Use your calculator to work out

$$\frac{\sqrt{46.2 - 17.5}}{2.39 \times 0.7}$$

Write down all the figures on your calculator display.

.....

(ii) Give your answer to (i) correct to 3 significant figures.

.....

(3)

(b) Work out $(2.34 \times 10^5) \times (5 \times 10^4)$

Give your answer in standard form.

.....

(2)

(Total for Question is 5 marks)

Q2.

Write the following numbers in order of size.
Start with the smallest number.

0.038×10^2 3800×10^{-4} 380 0.38×10^{-1}

.....

(Total for Question is 2 marks)

Q3.

(a) Simplify $x^7 \times x^3$

.....

(1)

(b) Simplify $(m^4)^3$

.....

(1)

(c) Simplify

$$\frac{36af^8}{12a^5f^2}$$

.....

(2)

(Total for Question is 4 marks)

Q4.

Pavel and Katie share some sweets in the ratio 3 : 8

Katie gets 32 sweets.

(a) How many sweets does Pavel get?

.....

(2)

Katie also has a tin of chocolates.

There are 80 chocolates in the tin. 45% of the chocolates have toffee in the middle.

(b) Work out the number of chocolates that have toffee in the middle.

.....

(2)

(Total for Question is 4 marks)

Q5.

(a) Simplify $4y + 2x - 3 + 3x + 8$

.....

(2)

(b) Factorise fully $9x^2 - 6xy$

.....

(2)

(c) Expand $4(x + 2)$

.....

(1)

(d) Expand and simplify $(x - 5)(x + 3)$

.....

(2)

(Total for Question is 7 marks)

Q6.

A cooker costs £650 plus 20% VAT.

(a) Calculate the total cost of the cooker.

£ . . . . . .

(3)

A washing machine has a price of £260
In a sale its price is reduced by £39

(b) Write the reduction as a percentage of the price.

. . . . . %

(2)

3 kitchen chairs cost a total of £44.79

(c) Work out the total cost of 8 of these chairs.

£ . . . . . .

(2)

(Total for Question is 7 marks)

Q7.

Write as a single fraction in its simplest form the result of subtracting

$\frac{7x-3}{x^2-1}$ from $\frac{2x}{x-1}$

.....

(Total for Question is 4 marks)

Q8.

Here are the first 5 terms of an arithmetic sequence.

3 9 15 21 27

(a) Find an expression, in terms of *n*, for the *n*th term of this sequence.

.....

(2)

Ben says that 150 is in the sequence.

(b) Is Ben right?
You must explain your answer.

.....
.....
.....

(1)

(Total for Question is 3 marks)

Q9.

Suppose

$$p^2 = \frac{x - y}{xy}$$

$x = 8.5 \times 10^9$
$y = 4 \times 10^8$

Find the value of p.
Give your answer in standard form correct to 2 significant figures.

.....

(Total for Question is 3 marks)

Q10.

Solve the simultaneous equations

$5x + 2y = 11$
$4x - 3y = 18$

$x =$ . . . . . .

$y =$ . . . . . .

(Total for Question is 4 marks)

Q11.
Colin, Dave and Emma share some money.

Colin gets $^{3}/_{10}$ of the money.

Emma and Dave share the rest of the money in the ratio 3 : 2

What is Dave's share of the money?

.....

(Total for Question is 4 marks)

Q12.
Solve

$$\frac{4x-1}{5} + \frac{x+4}{2} = 3$$

x = ...

(Total for Question is 3 marks)

Q13.
XYZ is a right-angled triangle.

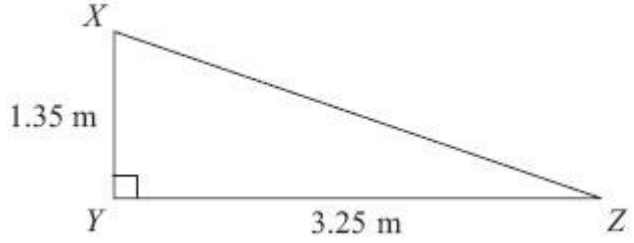


Diagram **NOT** accurately drawn

Calculate the length of *XZ*.
Give your answer correct to 3 significant figures.

.....

(Total for Question is 3 marks)

Q14.

Make t the subject of the formula $2(d - t) = 4t + 7$

$t =$ . . . . . .

(Total for Question is 3 marks)

Q15.

(a) (i) Factorise $x^2 - 12x + 27$

.....

(ii) Solve the equation $x^2 - 12x + 27 = 0$

.....

(3)

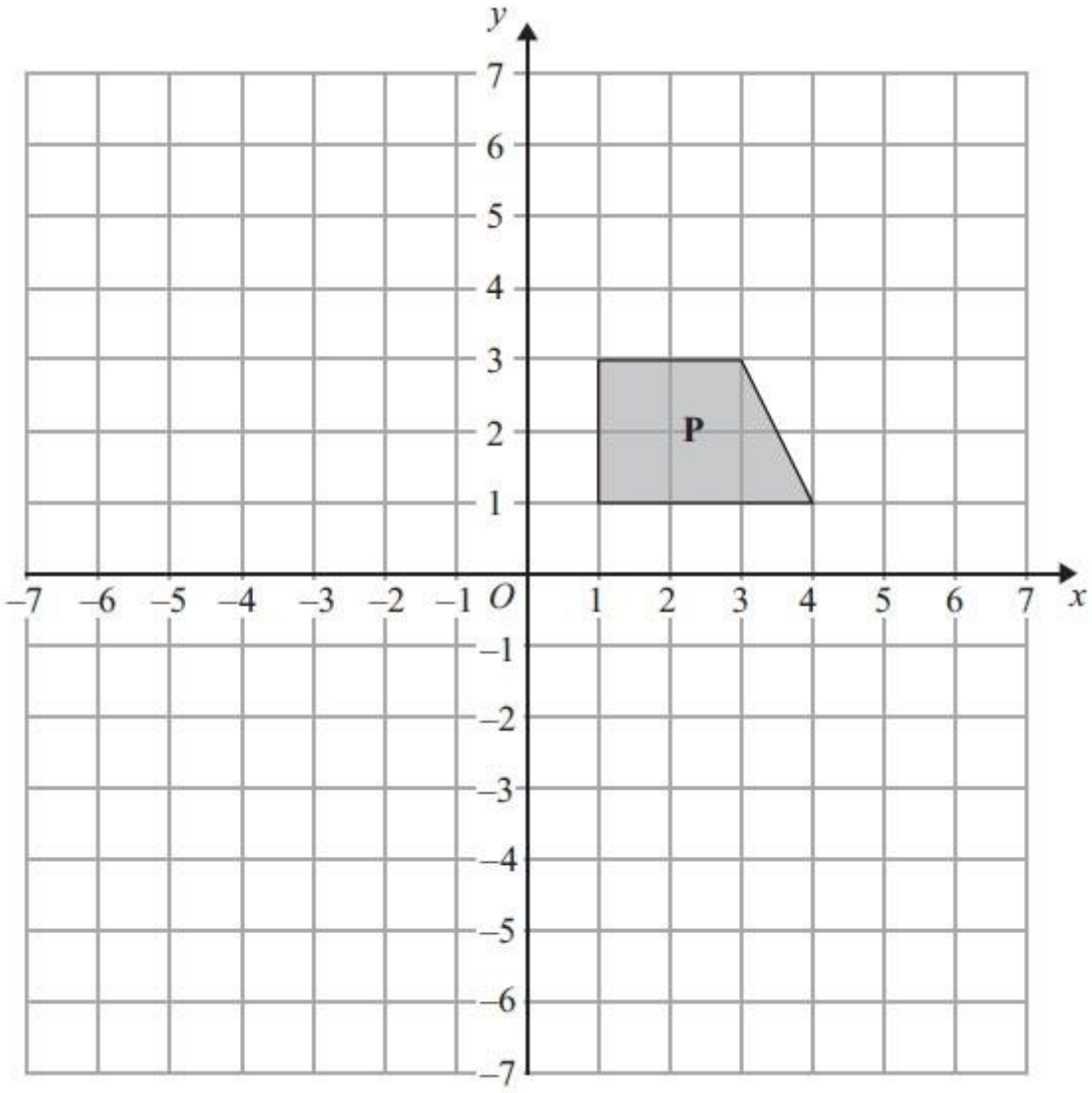
(b) Factorise $y^2 - 100$

.....

(1)

(Total for Question is 4 marks)

Q16.



Shape **P** is reflected in the line $x = -1$ to give shape **Q**.

Shape **Q** is reflected in the line $y = 0$ to give shape **R**.

Describe fully the **single** transformation that maps shape **P** onto shape **R**.

.....

.....

(Total for Question is 3 marks)

Q17.

Mr Watkins needs to buy some oil for his central heating.

Mr Watkins can put up to 1500 litres of oil in his oil tank.
There are already 850 litres of oil in the tank.
Mr Watkins is going to fill the tank with oil.

The price of oil is 67.2p per litre.
Mr Watkins gets 5% off the price of the oil.

How much does Mr Watkins pay for the oil he needs to buy?

.....

(Total for Question is 5 marks)

Q18.

Here are two triangles $\mathbf{T_1}$ and $\mathbf{T_2}$.

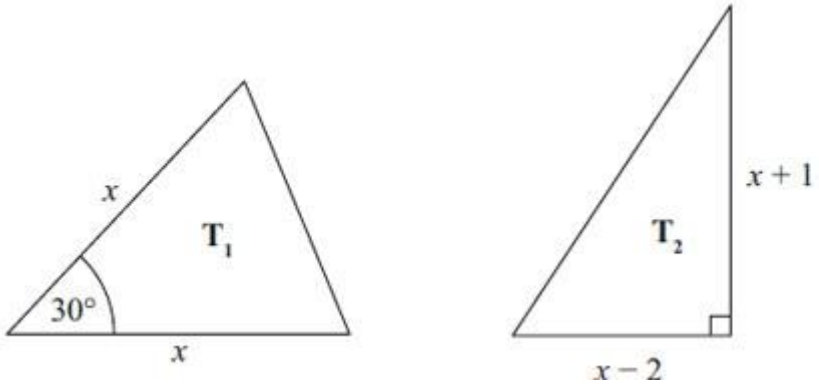


Diagram **NOT** accurately drawn

The lengths of the sides are in centimetres.

The area of triangle **T1** is equal to the area of triangle **T2**.

Work out the value of *x*, giving your answer in the form $a + \sqrt{b}$ where *a* and *b* are integers.

.....

(Total for Question is 5 marks)

Q19.

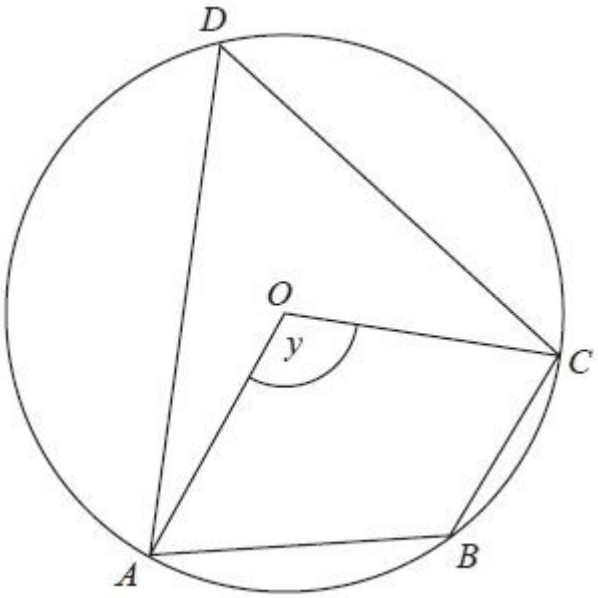


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.

Angle *AOC* = *y*.

Find the size of angle *ABC* in terms of *y*.

Give a reason for each stage of your working.

(Total for Question is 4 marks)

Q20.

(a) Solve

$$\frac{4(8x-2)}{3x} = 10$$

.....

(3)

(b) Write as a single fraction in its simplest form

$$\frac{2}{tan\theta + 3} - \frac{1}{tan\theta - 6}$$

.....

(3)

(Total for Question is 6 marks)

Q21.

Henry is thinking about having a water meter.

These are the two ways he can pay for the water he uses.

Water Meter	No Water Meter
A charge of £28.20 per year **plus** 91.22p for every cubic metre of water used **1 cubic metre = 1000 litres**	A charge of £107 per year

Henry uses an average of 180 litres of water each day.

Henry wants to pay as little as possible for the water he uses.

Should Henry have a water meter? (Show your working)

(Total for Question is 5 marks)

Q22.

Simplify fully

$$\frac{x^2-2x-15}{x^2-4x-21}$$

.....

(Total for Question is 3 marks)

Q23.
The diagram shows a large tin of pet food in the shape of a cylinder.

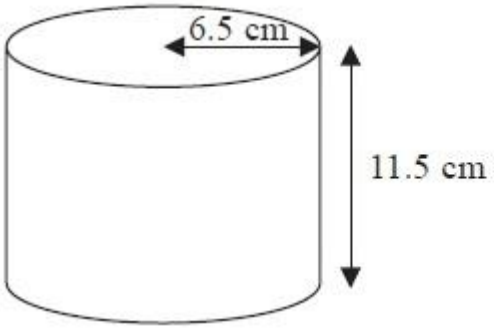


Diagram **NOT** accurately drawn

The large tin has a radius of 6.5 cm and a height of 11.5 cm.

A pet food company wants to make a new size of tin.

The new tin will have a radius of 5.8 cm.

It will have the same volume as the large tin.

Calculate the height of the new tin.

Give your answer correct to one decimal place.

..... cm

(Total for Question is 3 marks)

Q24.

Bob asked each of 40 friends how many minutes they took to get to work.

The table shows some information about his results.

Time taken (*m* minutes)	**Frequency**
$0 < m \leq 10$	3
$10 < m \leq 20$	8
$20 < m \leq 30$	11
$30 < m \leq 40$	9
$40 < m \leq 50$	9

Work out an estimate for the mean time taken.

. . . . . minutes

(Total for Question is 4 marks)

Q25.

The diagram shows a quadrilateral *ABCD*.

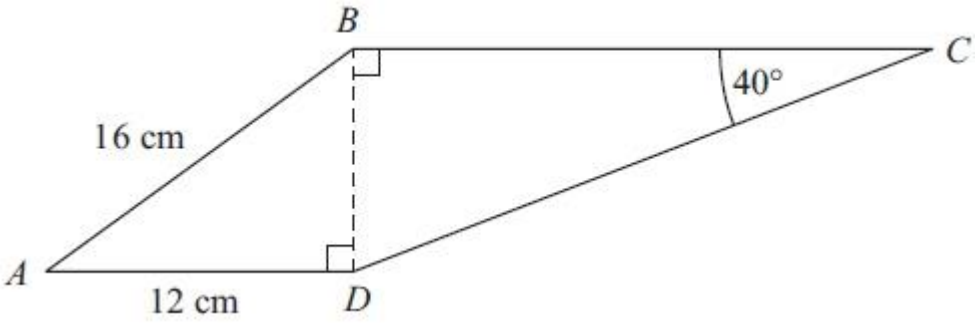


Diagram **NOT** accurately drawn

AB = 16 cm.
AD = 12 cm.
Angle *BCD* = 40°.
Angle *ADB* = angle *CBD* = 90°.

Calculate the length of *CD*.
Give your answer correct to 3 significant figures.

. . . . . cm

(Total for Question is 5 marks)

Q26.

Solve the simultaneous equations $x^2 + y^2 = 9$

$x + y = 2$

Give your answers correct to 2 decimal places.

x = ... y = ...

or x = ... y = ...

(Total for Question is 6 marks)

Q27.

Carolyn has 20 biscuits in a tin.

She has

12 plain biscuits
5 chocolate biscuits
3 ginger biscuits

Carolyn takes at random two biscuits from the tin.

Work out the probability that the two biscuits were **not** the same type.

.....

(Total for Question is 4 marks)

Q28.

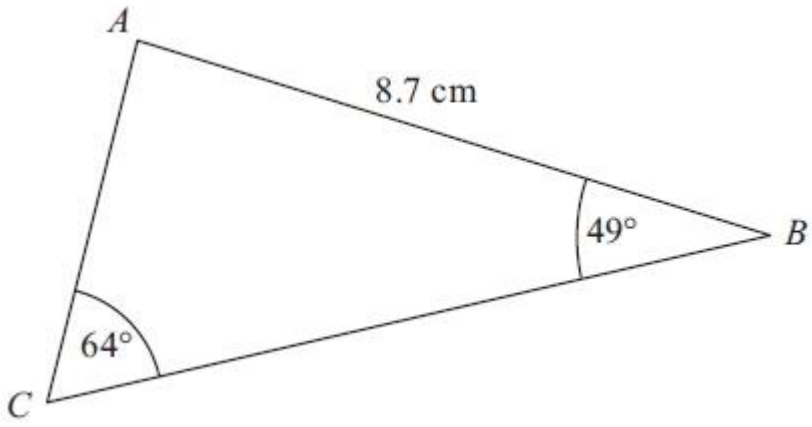


Diagram **NOT** accurately drawn

ABC is a triangle.

AB = 8.7 cm.
Angle *ABC* = 49°.
Angle *ACB* = 64°.

Calculate the area of triangle *ABC*.
Give your answer correct to 3 significant figures.

. . . . . cm^2

(Total for Question is 5 marks)

END OF EXAM